Separation of Delayed, Parameterized and Correlated Sources

H. Mortada[1], V. Mazet[1], C. Soussen[2], C. Collet[1]

[1]ICUBE, Université de Strasbourg - CNRS,
[2]CRAN, Université de Lorraine - CNRS

GDR-ISIS
Paris

20/03/2017

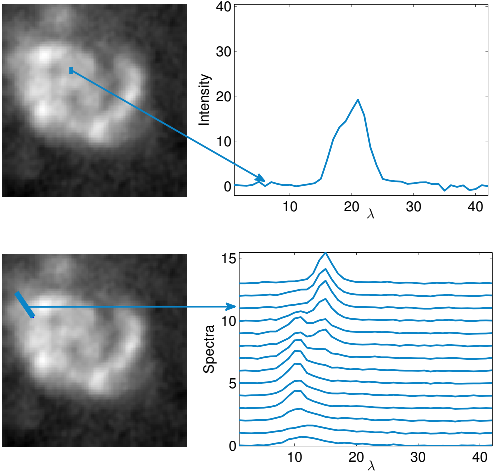
1. Introduction
   - Galaxy Kinematics
   - Goal
2. Proposed Method
   - Problem Formulation
   - Alternating Least Squares
   - Delay and Amplitude Estimation
   - Numerical results
3. Conclusion

Galaxy Kinematics

Spiral Galaxy M51-Nasa.gov

- Study of the internal gas motions
- Multiple structures

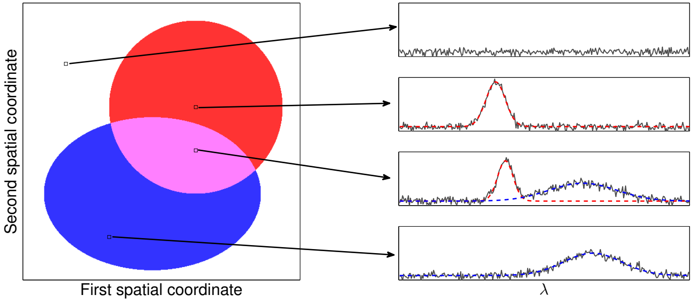
Galaxy Kinematics



Galaxy NGC 4254

Goal

- Each structure is attributed to a peak
- Varying characteristics through the image

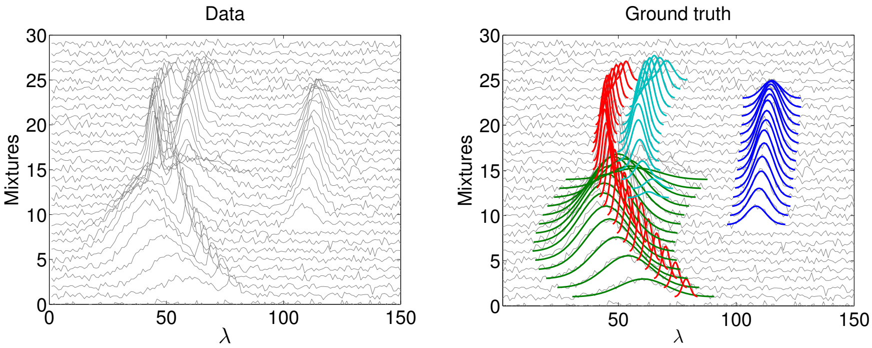


1 Introduction
Galaxy Kinematics
Goal

2 Proposed Method
Problem Formulation
Alternating Least Squares
Delay and Amplitude Estimation
Numerical results

3 Conclusion

- Spectra i are assigned to the mixtures
- Peaks j are assigned to the sources
- Parameter estimation and peak tracking are done simultaneously

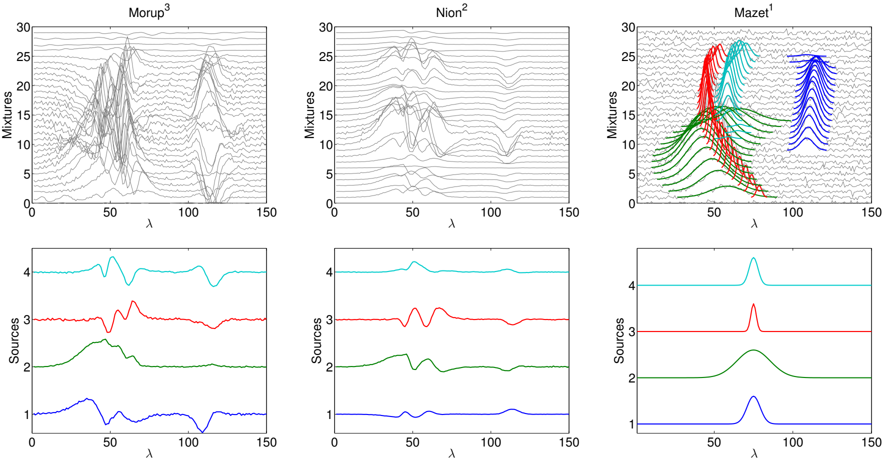


$I = 30$ mixtures $J = 4$ sources

Challenge: Correlated Sources

Strong assumptions: independent, uncorrelated or orthogonal sources

Bayesian method

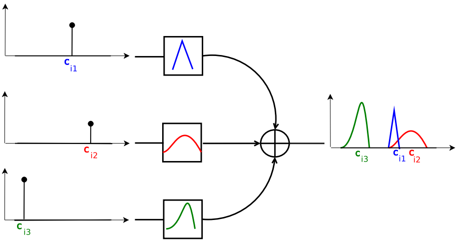


[1] Mazet *et al.*, *Signal Processing*, 2015
[2] Nion *et al.*, *LVA/ICA*, 2010
[3] Morup *et al.*, *ICA*, 2007

Problem Formulation

$$\boldsymbol{x}_i(\lambda) = \sum_{j=1}^{J} a_{ij} \boldsymbol{s}_j(\lambda - c_{ij}; w_j) + \boldsymbol{n}_i(\lambda) \quad \forall i$$

- Assumption: parameterized sources



- Special case: same waveform for all the sources

Problem Formulation

Estimate $\boldsymbol{A}$, $\boldsymbol{C}$ and $\boldsymbol{w}$ that minimize the residual error:

$$E(\boldsymbol{A}, \boldsymbol{C}, \boldsymbol{w}) = \sum_{i=1}^{I} \left\| \boldsymbol{x}_i(\boldsymbol{\lambda}) - \sum_{j=1}^{J} a_{ij} \boldsymbol{s}(\boldsymbol{\lambda} - c_{ij}; w_j) \right\|_2^2$$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & \dots & a_{1J} \\ \vdots & \ddots & \vdots \\ a_{I1} & \dots & a_{IJ} \end{bmatrix} \quad \boldsymbol{C} = \begin{bmatrix} c_{11} & \dots & c_{1J} \\ \vdots & \ddots & \vdots \\ c_{I1} & \dots & c_{IJ} \end{bmatrix} \quad \boldsymbol{w} = \begin{bmatrix} w_1 & \dots & w_J \end{bmatrix}$$

Alternating Least Squares

ALS Scheme:

Until convergence:

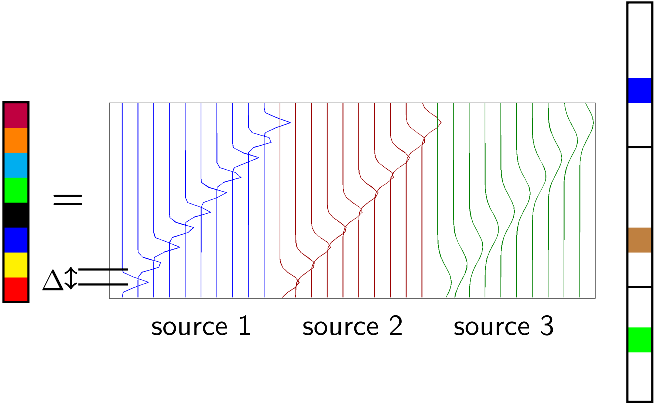
1. minimize $E(\boldsymbol{A}, \boldsymbol{C}, \boldsymbol{w})$ w.r.t. $\boldsymbol{w}$
   - Levenberg-Marquardt algorithm
2. minimize $E(\boldsymbol{A}, \boldsymbol{C}, \boldsymbol{w})$ w.r.t. $\boldsymbol{A}$ and $\boldsymbol{C}$
   - Parametric dictionary
   - OMP-like implementation

Delay and Amplitude Estimation

- Separable problem:

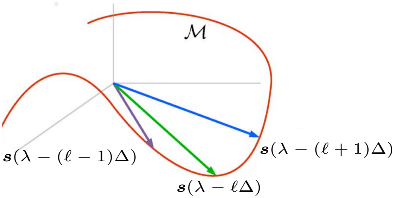
$$\min_{\boldsymbol{A},\boldsymbol{C}} E\big(\boldsymbol{A},\boldsymbol{C},\boldsymbol{w}\big) \quad \Leftrightarrow \quad \min_{\boldsymbol{a}_i,\boldsymbol{c}_i} \left\| \boldsymbol{x}_i(\boldsymbol{\lambda}) - \sum_{j=1}^{J} a_{ij}\,\boldsymbol{s}(\boldsymbol{\lambda} - c_{ij}; w_j) \right\|_2^2 \ \forall i$$

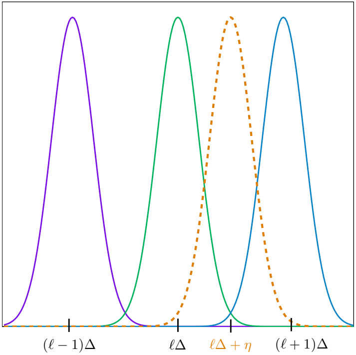
- Delay sampling step: Δ



Continuous Delay Estimation

- Translation-invariant signals
- interpolation strategy: polar[4,5]
- $c_{ij} = \ell\Delta + \eta \quad \ell \in \mathbb{Z},\ |\eta| < \Delta/2$

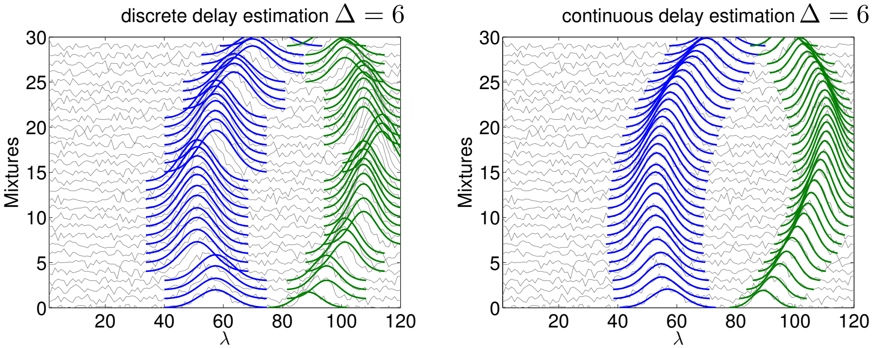




[4] Ekanadham *et al.*, *IEEE Trans. Signal Process.*, 2011

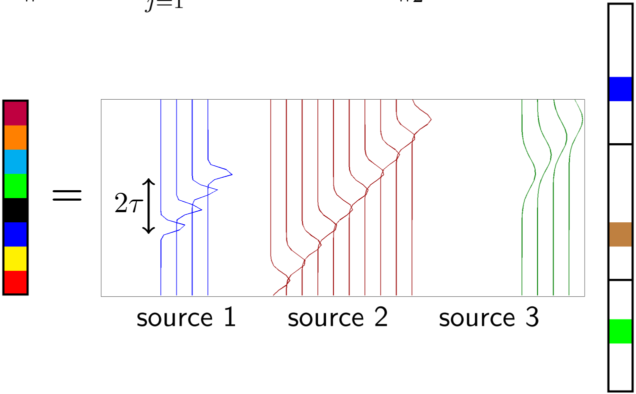
[5] Fyhn *et al.*, *IEEE Trans. Signal Process.*, 2015

Continuous Delay Estimation

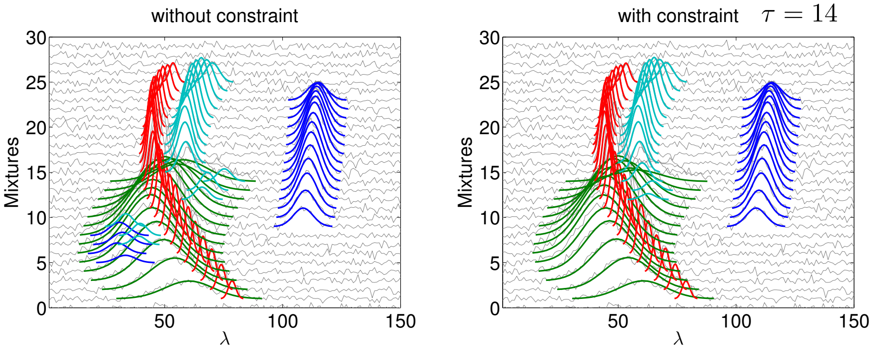


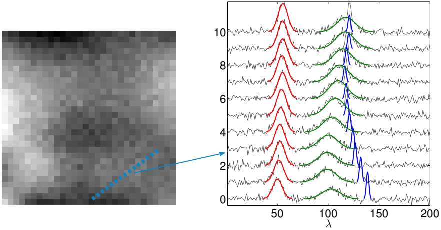
Slow Delay Evolution Constraint

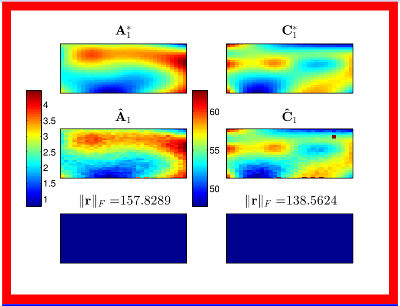
- Spatial neighboring information
- $\bar{c}_{ij} = \frac{1}{\text{card}(\mathcal{V})} \sum_{v \in \mathcal{V}} c_{vj}$
- $\min_{\boldsymbol{a}_i, \boldsymbol{c}_i} \left\| \boldsymbol{x}_i(\boldsymbol{\lambda}) - \sum_{j=1}^{J} a_{ij} \boldsymbol{s}(\boldsymbol{\lambda} - c_{ij}; w_j) \right\|_2^2$ s.t. $|c_{ij} - \bar{c}_{ij}| \leq \tau$

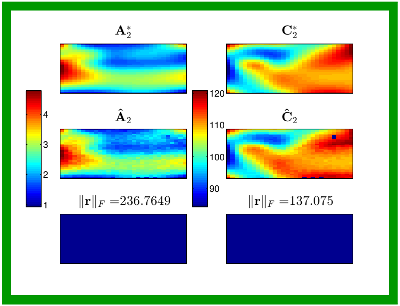


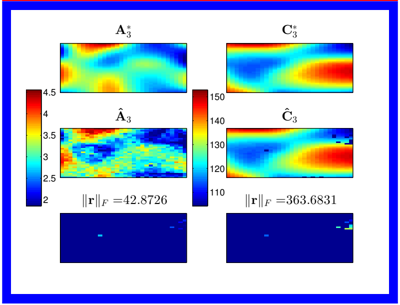
Slow Delay Evolution Constraint




10
8
6
4
2
0
50
100
150
200
λ

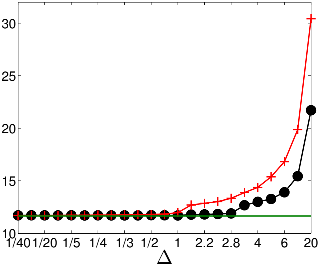

$\mathbf{A}_1^*$
$\mathbf{C}_1^*$
$\hat{\mathbf{A}}_1$
$\hat{\mathbf{C}}_1$
4
3.5
3
2.5
2
1.5
1
60
55
50
$\|\mathbf{r}\|_F$ =157.8289
$\|\mathbf{r}\|_F$ =138.5624


$\mathbf{A}_2^*$
$\mathbf{C}_2^*$
$\hat{\mathbf{A}}_2$
$\hat{\mathbf{C}}_2$
4
3
2
1
120
110
100
90
$\|\mathbf{r}\|_F$ =236.7649
$\|\mathbf{r}\|_F$ =137.075

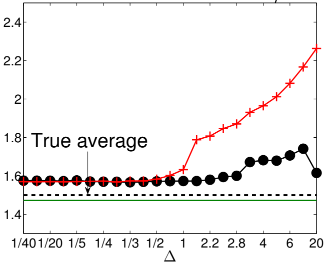

$\mathbf{A}_3^*$
$\mathbf{C}_3^*$
$\hat{\mathbf{A}}_3$
$\hat{\mathbf{C}}_3$
4.5
4
3.5
3
2.5
2
150
140
130
120
110
$\|\mathbf{r}\|_F$ =42.8726
$\|\mathbf{r}\|_F$ =363.6831

Numerical results

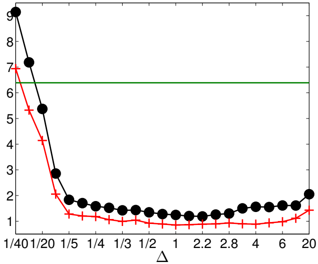
Residual norm E



Peak number $/I$



Comp. time (seconds)



+ Discrete version

• Continuous version

– MCMC

1 Introduction
- Galaxy Kinematics
- Goal

2 Proposed Method
- Problem Formulation
- Alternating Least Squares
- Delay and Amplitude Estimation
- Numerical results

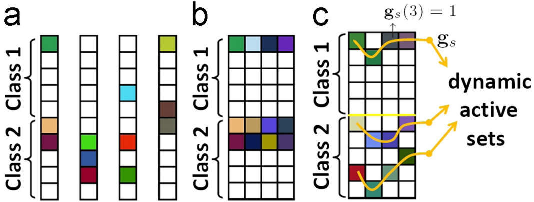
3 Conclusion

Contributions

- Delayed source separation model
- Parametrized and correlated sources
- Continuous delay estimation
- Constraint to ensure slow delay evolution
- As effective as the best competitors with much better computation time

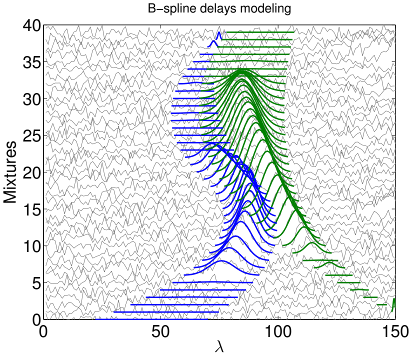
Perspectives

- Varying source shape through the mixtures
- Dynamic joint sparse representation [6] + smooth regularization



[6] Zhang *et al.*, *Pattern Recognition*, 2012

- B-spline trajectories modeling
- $\hat{\boldsymbol{U}} = \underset{\boldsymbol{U}}{\text{argmin}} \sum_{i=1}^{I} \left\| \boldsymbol{x}_i(\boldsymbol{\lambda}) - \sum_{j=1}^{J} \boldsymbol{s}\left(\boldsymbol{\lambda} - \sum_{k=1}^{K} u_{k,j} \boldsymbol{b}_{k,p}(i); w_{ij}\right) \right\|_2^2$
- $\hat{\boldsymbol{C}} = \boldsymbol{B}\hat{\boldsymbol{U}}$



References

Thank you!

- E. Villeneuve and H. Carfantan, Nonlinear deconvolution of hyperspectral data with MCMC for studying the kinematics of galaxies, *IEEE Trans. Image Process.*, 4322–4335, 2014.
- J. Duan. Restoration and separation of piecewise polynomial signals. Application to Atomic Force Microscopy. Phd thesis, Université Henri Poincaré, Nancy, 2010.
- C. Ekanadham, D. Tranchina, and E. P. Simoncelli. Recovery of sparse translation-invariant signals with continuous basis pursuit. *IEEE Trans. Signal Process.*, 59 :4735–4744, 2011.
- K. Fyhn, M. F. Duarte, and S. H. Jensen. Compressive parameter estimation for sparse translation-invariant signals using polar interpolation. *IEEE Trans. Signal Process.*, 63 :870–881, 2015.
- V. Mazet, S. Faisan, S. Awali, M.-A. Gaveau, and L. Poisson. Unsupervised joint decomposition of a spectroscopic signal sequence. *Signal Process.*, 109 :193–205, 2015.
- M. Mørup, K. H. Madsen, and L. K. Hansen. Shifted independent component analysis. In *ICA*, 2007.
- D. Nion, B. Vandewoestyne, S. Vanaverbeke, K. Abeele, H. Gersem, and L. De Lathauwer. A time-frequency technique for blind separation and localization of pure delayed sources. In *LVA/ICA*, 2010.

mortada@unistra.fr