Affine Transformations of Digital Images using Arbitrary Precision Computations

Areas of Research : Discrete Geometry and Formal Proofs

Internship position (spring 2015)

Host Research Lab.

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Keywords : discrete geometry, discrete affine transformation, Sage, Python, formal proofs, Coq

Presentation

Using affine or linear transformations such as rotations, symmetries, homotheties and shear mappings is commonplace in digital image processing. These transformations are usually modelled as matrices of real numbers. Because floating-point numbers can not represent all real numbers, but only a finite set of rational numbers, the implementations of affine transformations often have properties which differ from those of the *real* transformations they are supposed to represent. This may be the case for algebraic properties (such as bijectivity), topologic ones (such as connexity), and geometric ones (such as distance conservation).

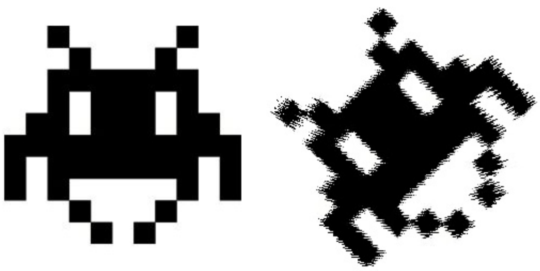

FIGURE 1 – Example of a discrete transformation (a rotation) of a digital image

Quasi-Affine Applications (a.k.a. by QAA) [CBC09] are obtained by the discretization of affine applications. They can be represented as matrices of rational numbers and allow to describe linear or affine transformations such as the discrete rotations [NKPT13] (see Fig. 1).

An Ω-*QAA* [ACF+14] can be viewed as a sequence of these Quasi-Affine Applications whose successive values (rational numbers) are closer to the actual *real* application. Using this representation allows both capturing the discrete features of the plane (or space) as well as computing exactly with a discrete model of the continuum based on integers only : the Harthong-Reeb line [CWF+12, MCF14].

In this internship, we shall state and prove the mathematical properties of the Ω-QAA as well as those of their implementation. We shall describe these applications formally using the Coq proof assistant [Coq14, BC04] and then prove formally their basic properties in this setting.

Several algorithms based on the Ω-QAA and implementing digital images transformations are being currently developed using Python/Sage. The first step will consist in carrying out experiments to state some properties of these transformations as conjectures, especially with respect to its connections with the *reference* real transformation. Next, we shall describe these algorithms in Coq and formally prove their properties.

A possible extension consists in adapting these algorithms to a three-dimensional setting as well as formalizing and proving the associated properties.

Références

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Funding

This internship position is funded by the research project TrADiCont : **Tr**ansformations **A**ffines **Di**scrètes pour la représentation du **Cont**inu (internal research project of the Icube lab. 2014-2015).

Requirements

Good programming skills are needed. The programming languages we shall use are Python, Ocaml and Sage. An extended knowledge in mathematics is also required. Some knowledge/experience in the field of discrete geometry and/or formal proofs using the Coq proof assistant would be appreciated, but is not required.