

Joint Decomposition using Sparse Approximation in Astronomical Multispectral Images

Context

Studying the gas kinematics within galaxies yields key information to understand the story and evolution of the Universe. Telescopes today routinely provide high-resolution multispectral images, *i.e.* 3D images where the third dimension is the wavelength : each pixel of the multispectral image is a peak (or line) spectrum. Then, the spectrum of galaxies is observed to measure the peak shift due to the Doppler effect and induced by galaxy kinematics.

The standard analysis of multispectral images remains the visual inspection and the use of low-level interactive procedures. An easier and sharper analysis requires efficient signal and image processing algorithms.

The problem comes to decompose each spectrum into a sum of peaks, each one described by some shape parameters (centre, width, amplitude). These parameters move slowly between two neighbouring spectra. Furthermore, the signal-to-noise ratio is often very low. For these reasons, it is essential to introduce the spatial redundancy so as to provide a reliable and robust processing with respect to noise. In other words, the decomposition must be performed jointly on all spectra.

Problem

The spectroscopic decomposition is considered as an inverse problem. Besides, a sparse regularization appears natural since the spectra are supposed to have a small number of peaks. As far as we know, there is no approach to perform a joint decomposition of the spectra, except our recent work on temporal sequences [1]. This approach is set in a Bayesian framework and uses MCMC algorithms, but its major shortcoming concerns the difficulty to properly explore the solution space in a reasonable computation time, especially because the unknowns are highly correlated.

For these reasons, we would like to investigate the latest techniques of sparse approximation (*e.g.* [2]). The idea is to model each spectrum \mathbf{y}_s as the product of an overcomplete “dictionary” \mathbf{A} and a vector \mathbf{x}_s with few non-zero elements. Each column of \mathbf{A} represents a potential peak in the spectrum. The estimation of \mathbf{x}_s is obtained by minimizing a criterion composed of a data fitting term and a term favouring sparsity (such as $\|\mathbf{x}_s\|_0$ or $\|\mathbf{x}_s\|_1$). In our context, we need two more terms.

First, the peak amplitudes \mathbf{x}_s are positive (emission lines). This non-negativity constraint impacts the choice of optimization algorithms, such as the non-negative version of OMP [3] or proximal algorithms whose structure is naturally consistent with the use of a positivity constraint (ADMM [4] or iterative thresholding algorithms [5, 6, 7]). A first objective of this thesis is to compare these algorithms in the context of ill-conditioned inverse problems.

The second term concerns the still open problem of joint decomposition of spectra through the inclusion of spatial redundancy (as the peaks evolve slowly). The methods for linking atoms together (group or structured sparsity [8]) cannot directly apply because the peak evolution is unknown and varies between two spectra. Moreover, many recent works (simultaneous sparsity [9]) jointly decompose data on a dictionary, but the underlying assumption is too strong to be applied here. Therefore we have to propose new solutions to link the vectors \mathbf{x}_s pairwise with more flexibility.

Finally, the proposed approaches will be applied to real astronomical images, whose sizes reach up to almost 4000 bands in the case of MUSE. It is therefore necessary to implement efficient methods to perform the decomposition in a reasonable time.

Required Skills

The candidate needs a master or engineering degree. He/She should have a strong background in mathematics and signal & image processing. He/She will know scientific programming (Matlab, Python, ...). Also, he/she should be open-minded and able to interact in a multidisciplinary context.

Funding

This PhD position will be funded by the French ANR (Agence nationale de la recherche) within the project DSIM (dsim.unistra.fr); it will begin in 2015.

Supervision

The supervision will be conducted by Vincent MAZET (ICube), Charles SOUSSEN (CRAN), and Christophe COLLET (ICube). It will take place in the MIV team at ICube laboratory (University of Strasbourg).

Applications

Please send your application to Vincent MAZET (vincent.mazet@unistra.fr) providing a CV, a covering letter, one (at least) reference and the last academic results (with rankings, where applicable).

References

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